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LETTER TO THE EDITOR

Novel A–B type oscillations in a 2D electron gas in inhomogenous magnetic fields

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Abstract. We present results from a quantum and semiclassical theoretical study of the ρ_{xy} and ρ_{xx} resistivities of a high mobility 2D electron gas in the presence of a dilute random distribution of tubes with magnetic flux Φ and radius R, for arbitrary values of $k_f R$ and $F = e\Phi/h$. We report on novel Aharonov–Bohm-type oscillations in ρ_{xy} and ρ_{xx}, related to degenerate quantum flux tube resonances, that satisfy the selection rule $(k_f R)^2 = 4F(n + \frac{1}{2})$, with n an integer. We discuss possible experimental conditions where these oscillations may be observed.

Transport in a two-dimensional electron gas (2DEG) in the presence of weak *inhomogenous* magnetic fields has recently been the subject of considerable interest, both experimental [1–3] and theoretical [4]. This situation has been achieved experimentally by gating the 2DEG system with a type-II superconducting layer. Abrikosov vortices are then produced by applying an external magnetic field perpendicular to the plane of the layers. In the *ballistic* transport regime and for low fields, when the density of vortices is small, clear modifications to the Hall resistance in the quantum regime have been measured [3]. Previous theoretical studies of this problem were restricted to the asymptotic semiclassical regime $k_f R \gg 1$ [4] and the quantum limit $k_f R \ll 1$ [5]. However, the experiments have covered the interesting intermediate $k_f R$ crossover regime, with $F = \frac{1}{2}$. Here, k_f is the Fermi wavevector and $F = \Phi/\Phi_0$ with $\Phi_0 = h/e$ the quantum flux.

In this letter we present a full solution to this problem for arbitrary $k_f R$ and F. Our results identify a series of novel quantum oscillations in the galvanomagnetic properties of the 2DEG, that appear to be within the reach of experimental confirmation. These oscillations can be seen at intermediate ranges of $k_f R$ and $F (> \frac{1}{2})$ values and are related to the Aharonov–Bohm (AB) effect. The intermediate ranges of $k_f R$ are already achievable experimentally (e.g. [1, 3]). At the end of the letter we discuss two experimental set-ups that have been suggested to produce larger values of F. Here we are interested in the experimental situation considered in [3] where the 2D electrons move ballistically between the flux tubes and the dominant transport mechanism can be assumed to come from electrons scattering off individual flux tubes. Under these conditions, as a first approximation, we can apply the results of linear response theory in the Born approximation [6]. These results are formally the same as those obtained with the Boltzmann equation [4]. The weak non-local localization limit has already been considered experimentally and theoretically [2].

There are three important physical contributions to the electronic transport properties of this system: (i) for finite R the Lorentz force that leads to an asymmetry in the scattering process; (ii) a diffractive force, relevant in the $0 < k_f R < 1$ regime and first considered by Iordanskiĭ [7], that also yields a transversal contribution to the transport, and (iii) the standard AB contribution [8]. The Iordanskiĭ term in ρ_{xy}, which is not taken into account in the differential cross section, is due to the scattering of electrons by finite radius flux tubes and has essentially the same origin as the AB effect [9], for both are topological in nature and due to the long-range properties of the vector potential. This means, as we see below, that the contribution from (ii) to the Hall resistance only depends on the value of F and not on the specific magnetic flux profile chosen in the analysis.

The modification to the Hall resistivity, ρ_{xy}, due to the inhomogenous field can be represented by a *Hall coefficient*, α, which is defined by the expression [3]

$$\rho_{xy} = \alpha(k_f R, F)\frac{B}{n_e e} \tag{1}$$

where n_e is the electron density and B the magnetic field. In the Born approximation of the Kubo formula the transport coefficients are expressed in terms of the scattering cross section $f(\phi)$, with ϕ the electronic scattering angle [6, 4]. Explicit limiting values of α have been calculated in the extreme quantum $\alpha(k_f R \ll 1) = (1/2\pi F)\sin(2F\pi)$ and semiclassical limits $\alpha(k_f R \gg 1) = 1$ [4, 5]. We use these two results as constraints to be satisfied in our calculations. Previous studies were restricted to these two limits because of mathematical difficulties in the evaluation of the scattering amplitude $f(\phi)$ in the whole $k_f R$ and F ranges. These difficulties were identified by Khaetskiĭ [4] and are essentially related to the singularity in the AB and Iordanskiĭ scattering in the forward direction. In the AB case $f_{AB}(\phi \sim 0) \sim 1/\sin(\phi/2)$, which would lead to an infinite α [5, 7].

Below we present the results of an explicit evaluation of $f(\phi)$ in the whole range of $k_f R$ and F values. More importantly, we use these results to calculate α and the magnetoresistance ρ_{xx} in the extended parameter range. Since the calculational problems arise in the forward scattering region we consider the regularized scattering amplitude

$$f_\epsilon(\phi) = \frac{e^{-i\pi/4}}{\sqrt{2\pi k_f}} \sum_{m=-\infty}^{\infty} e^{im\phi}\, e^{-|m|\epsilon}\, [e^{2i\delta_m} - 1] \tag{2}$$

with ϵ the regularization parameter, which is taken to zero at the end of the calculations. Here δ_m is the phase shift associated with the mth partial wave and can be written as $\delta_m = \delta_m^{AB} - \tilde{\delta}_m$. The $\delta_m^{AB} = \frac{1}{2}\pi(|m| - |m + F|)$ accounts for the AB phase shift, and $\tilde{\delta}_m = \tan^{-1}(b_m/a_m)$ for the remaining contribution to the scattering. The coefficients a_m and b_m are obtained from the asymptotic wave function solution to the Schrödinger equation $\psi(r \to \infty, \theta) \approx \sum_{m=-\infty}^{\infty} [a_m J_\nu(k_f r) + b_m N_\nu(k_f r)]\, e^{im\theta}$, which has the required form for incoming plane waves and outgoing circular waves. Here $J_\nu(k_f r)$ and $N_\nu(k_f r)$ are the Bessel and Neumann functions of order $\nu = |m + F|$ (see [10] for more details).

Including the contributions (i)–(iii) we can then write the Hall coefficient as,

$$\alpha = \frac{k_f}{2\pi F} \lim_{\epsilon \to 0} \int_0^{2\pi} \sin\phi\, |f_\epsilon(\phi)|^2\, d\phi + \frac{1}{\pi F}\sin(F\pi). \tag{3}$$

The first term is the regularized Boltzmann contribution while we wrote the second topological term following Iordanskiĭ [7]. An important property of this expression is that it fully reduces to the extreme quantum and semiclassical results mentioned above. In this expression, as long as ϵ is finite, there is no singularity in $f_\epsilon(\phi)$ and we can perform the integral. After evaluating this integral using (2), and from the general fact that $[\delta_{m+1} - \delta_m] \to 0$ in the limit $|m| \to \infty$, we get the *finite* result

$$\alpha = -\frac{1}{2\pi F} \sum_{m=-\infty}^{+\infty} \sin [2(\delta_{m+1} - \delta_m)] . \tag{4}$$

This equation is one of the main results of this letter, for it provides an algorithm to calculate α for arbitrary values for the parameters $k_f R$ and F. The number of terms needed in the sum will depend on the parameter range considered.

Another way of regularizing the divergence in $f(\phi)$, which we used as a further check of the validity of our results, is by writing

$$\alpha = \frac{k_f}{2\pi F} \lim_{\epsilon' \to 0} \int_{\epsilon'}^{2\pi} \sin\phi \, |f_{\epsilon=0}(\phi)|^2 \, \mathrm{d}\phi + \frac{1}{2\pi F} \sin(2F\pi) . \tag{5}$$

Both expressions given in (3) and (5) lead to the same numerical results for α. Note that (3) and (5) differ in an important way from the expressions obtained before [4, 5], in that both expressions have an extra topological term $(1/\pi F)\sin(F\pi)$ and $(1/2\pi F)\sin(2F\pi)$, respectively. The reason the additive terms are different is that the $\epsilon \to 0$ and $\epsilon' \to 0$ limits do not commute, which shows a link between the regularization of the singularity at the origin and the Iordanskiĭ force.

The corresponding linear response theory result for the magnetoresistance $\Delta\rho_{xx}$ is

$$\frac{\Delta\rho_{xx}(F)}{\rho_{xx}(0)} = \frac{\tau_i}{\tau} = N_{\mathrm{F}}\ell_i \lim_{\epsilon \to 0} \int_0^{2\pi} (1 - \cos\phi) \, |f_\epsilon(\phi)|^2 \, \mathrm{d}\phi \tag{6}$$

$$= \frac{2}{k_f} N_{\mathrm{F}}\ell_i \sum_{m=-\infty}^{+\infty} \sin^2(\delta_{m+1} - \delta_m) \tag{7}$$

where τ is the time between electronic flux tube collisions, within the Kubo–Born approximation, and N_{F} the concentration of magnetic flux tubes. When the magnetic field is zero the transport scattering time is $\tau_i = \ell_i / v_f$, with v_f the Fermi velocity and ℓ_i the impurity scattering elastic mean free path, which is assumed to be much larger than $2R$.

We now proceed to present our results from the direct evaluation of (4) and (6). Later in this letter we present a physical explanation of these results using a semiclassical analysis. In plotting these results we have used the typical experimental parameter values given in the captions. In figure 1(*a*) we show α as a function of $k_f R$ for different values of F. The curve $F = \frac{1}{2}$ corresponds to an extended range of figure 3 in [3]. We note that for values of $F \leqslant 1$, α is a monotonic function of $k_f R$. Notice, however, that for $F = \frac{3}{4}$, α can become negative for small values of $k_f R$, which comes from our careful treatment of the extreme quantum region. For $F \geqslant 2$ we see clear oscillations in the α versus $k_f R$ curves [14]. For $F = 10$, for example, we can clearly identify sharp oscillations of α versus $k_f R$, although their absolute value is smaller. The number of oscillations as a function of $k_f R$ is equal to the integer part of F, $[F]$. For $F = 10$, there are ten oscillations (five of them not shown occur for $k_f R > 15$). Moreover, for small $k_f R$ there are narrower oscillations superimposed on the first few oscillations. In figure 1(*b*) we show the Hall resistivity as a function of F and $k_f R = 10$. For small values of F we see the classical linear behaviour of ρ_{xy} up to a maximum value, after which it decreases as F increases. We note that the quantum curve, obtained using (4), decreases non-monotonically as F increases and even becomes negative for values of $F \sim 50$. Finally, in figures 2(*a*) and 2(*b*) we show the corresponding results for $\Delta\rho_{xx}(F)/\rho_{xx}(0)$ as a function of both $k_f R$ and F. In figure 2(*a*) for $F \leqslant 1$ we see that $\Delta\rho_{xx}(F)/\rho_{xx}(0)$ is a monotonic decreasing function of $k_f R$, while for larger values of F

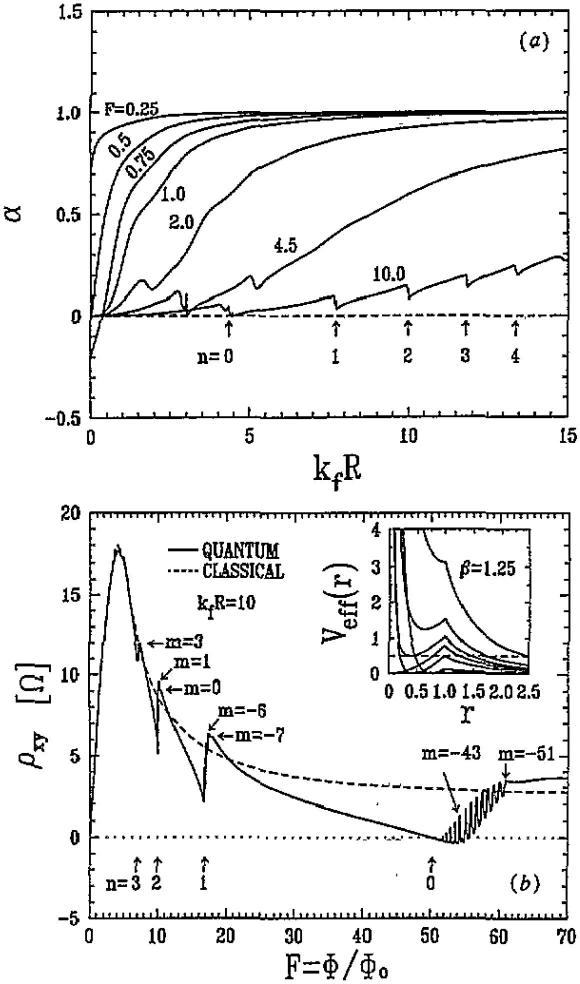

Figure 1. (*a*) Hall coefficient α as a function of k_fR. Here $R = 100$ nm, the density of flux tubes is $N_F = 10^5$ mm^{-2} and the electron concentration is $n_e = 3.98 \times 10^{10}$ cm^{-2}, with $\Phi = \pi R^2$. Here n denotes the flux tube resonances and m the angular momentum degeneracies. The broken curve indicates the zero value for α. (*b*) same as in (*a*) for the Hall resistivity ρ_{xy} as a function of F. The broken curve corresponds to the classical results. The inset shows the effective potential V_{eff} for different values of the total angular momentum (from top to bottom $J = 1.25, 0.5, 0.2, 0, -0.25, -0.75$).

the magnetoresistance becomes an oscillatory function of k_fR. In figure 2(*b*) we note the sharp resonances that occur exactly at the same values of the minima in ρ_{xy}.

We now provide a physical interpretation of these results in the semiclassical limit. To understand the semiclassical analysis, we start by discussing the classical problem [15]. We note that the energy, $E = \frac{1}{2}m^* v_f^2$, and the total (particle + field) angular momentum, $J = m^* v_f b - e\Phi$, are constants of the motion. Here b is the impact parameter and m^* the electron's effective mass. The impact parameter is defined as $b = y(t \to -\infty)$, where y is along the perpendicular direction of the current. The *classical* Hall coefficient is characterized by the important parameter $\beta \equiv \omega_c T = e\Phi/(2\pi m^* v_f R) = F/k_fR$. Different scattering events have different total angular momenta and different β parameters. When $\beta \ll 1$, the electron trajectories are only slightly affected by the magnetic flux tube. As β increases the Lorentz force becomes important until a critical β_c, above which trapped orbits can exist. The particular quantitative value of β_c depends upon the particular flux tube profile. For our flux-tube model $\beta_c = \frac{1}{2}$, and the trapped circular orbits have radius $r_0 = R/2\beta$. Both the quantum and classical scattering problems can be separated into angular and radial components. The radial component of the classical equation is, as usual, a

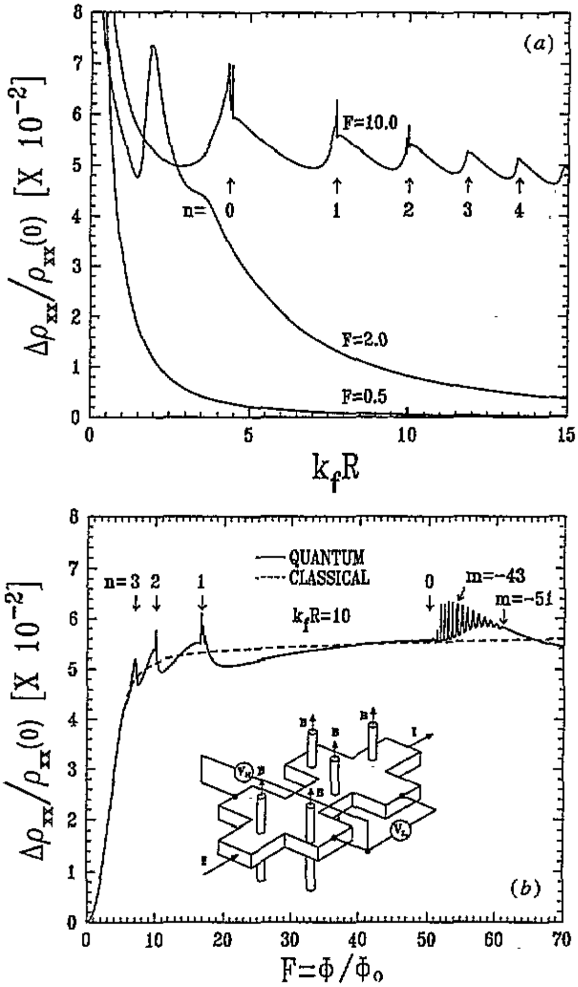

Figure 2. $\Delta\rho_{xx}(F)/\rho_{xx}(0)$ versus $k_f R$ (*a*) and versus F (*b*) for the same parameter values as in figure 1, with mean free path $\ell_i = 2\,\mu$m. The inset shows the system considered in this letter formed by a Hall bar with a dilute random distribution of perpendicular magnetic flux tubes of strength $F\Phi_0$. See text for further details.

one-dimensional problem with effective potential, $V_{\rm eff}(r,b) = \left[J + \beta \int_0^r r' B(r')\,{\rm d}r'\right]^2/2r^2$. Here the magnetic field of the flux tube is $\boldsymbol{B}(r) = B(r)\boldsymbol{z}$, and we have rescaled energies by $m^* v_f^2$, the angular momentum by $2\pi m v_f R = h k_f R$, and distances by R. In these units $J = b - \beta$ and the flux tube radius is equal to 1. In the inset of figure 1(*b*) we show curves for $V_{\rm eff}(r,b)$ for different values of J. We see that electrons with different Js, or impact parameters, experience different effective potentials. As the total angular momentum decreases, $V_{\rm eff}(r,b)$ develops a potential barrier with height $[J+\beta]^2/2$, which decreases rapidly. We can show that for $\beta \geqslant \beta_c = \frac{1}{2}$ there is a range of Js for which there can be trapped circular orbits inside the flux tube from $J_1 = 1/4\beta$ decreasing to $J_2 = 1 - \beta$. As J decreases from J_1 to J_2, the centre of the electronic circular orbit shifts from $r_1 = 0$ to $r_2 = 1 - r_0 = 1 - 1/2\beta$. This range of possible total angular momenta is such that the circular orbit stays completely within the flux tube. Classically, these circular orbits cannot be reached by a scattering process. However, quantum mechanically the scattering electron can tunnel through this potential barrier and form a quasi-bound state inside the flux tube for a finite time, and then escape again. In the classical calculation of the Hall resistivity and magnetoresistivity shown by broken curves in the figures, we computed the classical differential cross-section which was used in (1), (3) and (6), in place of $|f(\phi)|^2$.

In the semiclassical analysis we associate a classical circular orbit to each quasi-bound state. Using the standard Bohr–Sommerfeld quantization condition it is easy to show that the quasi-bound states are degenerate and occur at quantized values of the energy, $E_n/\hbar\omega_c = (k_fR)^2/4F = n + \frac{1}{2}$, with n an integer. This result is the analogue of the Landau level condition in a homogeneous magnetic field. The factor $n + \frac{1}{2}$ gives the total number of flux quanta enclosed by the circular orbit. Since the quanta of flux in the tube is equal to F, the quantum number n ranges from 0 to $[F] - 1$. Moreover, the quantized total angular momentum, $J_m = m\hbar$, leads to a degeneracy of the n levels. This degeneracy is equal to the total number of quantized circular orbits which we can put inside the flux tube. From the range of classically allowed circular orbits mentioned above, we deduce that the allowed m values start at $m_1 = [(k_fR)^2/4F] = n$ and decrease down to $m_2 = [k_fR - F] + \delta$, with $\delta = 1$ if $[k_fR - F] > 0$ and $\delta = 0$ if $[k_fR - F] \leqslant 0$. Therefore, we conclude that the degeneracy is equal to $m_1 - m_2 + 1 = n + 1 + [F - k_fR] - \delta$. In the figures, the arrows indicate the position of the principal quantum number n calculated using the selection rule $(k_fR)^2 = 4F(n + \frac{1}{2})$. We observe that they are remarkably well aligned with some maxima and minima of ρ_{xx} and ρ_{xy}. Furthermore, we have numerically determined that each resonance in ρ_{xx} and ρ_{xy} occurs at a preferential angular momentum $J_m = m\hbar$ for some m. We arrived at this conclusion by evaluating the time delay $t_D^m(k_fR, F) = 2\hbar(\partial\delta_m/\partial E) = (2R/v_f)[\partial\delta_m/\partial(k_fR)]$ and we found that as a function of m, t_D^m becomes sharply peaked for one particular value of m each time the pair (k_fR, F) corresponds to a resonance in the transport coefficients. In the figures we have indicated the values of m for the resolved resonances. The minima and maxima in ρ_{xy} correspond to quasi-bound states due to the tunnelling of the electron into the flux tube. Semiclassically, as m decreases the resonances correspond to rotationally asymmetric orbits leading to larger ρ_{xy}, as observed in figure 1(b). Note that the number of resonances observed for a particular quasi-bound state level should be equal to the degeneracies of this level. However, near m_1, the amplitude of the resonances is suppressed due to the exponentially small tunnelling probability through the potential barrier $(= [J + \beta]^2/2)$. For example note that $\rho_{xy} = 0$ at $F = 50$ in figure 1(b). On the other hand, for each quasi-bound state level, the observed resonances with the smallest m ($m = 3, 0, -7, -51$ in figure 1(b)) correspond precisely to the value m_2 derived semiclassically.

We now consider the possible experimental conditions necessary to observe the galvanomagnetic oscillations described in this letter. The variation of k_fR in the ranges of interest has already been achieved [1, 3]. New techniques need to be developed to produce larger values of F inside the flux tubes. We discuss a couple of possibilities that have already been suggested to us. The general idea is to have the usual Hall bar shown schematically in the inset of figure 2(a), with the inhomogenous magnetic field produced by a dilute distribution of magnetic flux tubes of strength F. One possible way to get larger values of F experimentally is by depositing randomly located submicron size *superconducting dots* or *pillars* on top of the 2DEG, in a manner similar to the way the dot and antidot systems have been fabricated [11, 12]. Alternatively, one may drill randomly located submicron holes in the superconducting layer by using electron-beam lithography [13]. In both cases, by following a magnetic field cooling technique the magnetic flux may be pinned inside the dots thus trapping a large bundle of flux quanta. As in the antidot systems we do not expect that the oscillations found here will be significantly affected by temperature or Coulomb effects, provided the temperatures are sufficiently low and the charging energy effects are not significant for the superconducting pillars fabricated.

In conclusion, we have presented a detailed analysis of the transport properties of a 2D electron-gas system in the presence of a dilute gas of randomly located magnetic flux tubes for arbitrary values of $k_f R$ and F. The main result from our analysis is the presence of novel AB-like oscillations in the galvanomagnetic properties of the system. These oscillations are explained in terms of the degenerate resonant levels, satisfying the selection rule $(k_f R)^2 = 4F(n + \frac{1}{2})$, due to the effective trapping potentials produced by the flux tubes. A more extensive presentation of the results described here will appear elsewhere [16].

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References

[1] Geĭm A K *et al* 1989 *Pis'ma Zh. Teor. Fiz.* **50** 359 (Engl. transl. 1989 *JETP Lett.* **50** 389)
Avishai Y and Band Y B 1991 *Phys. Rev. Lett.* **66** 1761
Kruithof G H *et al* 1991 *Phys. Rev. Lett.* **67** 2725
Geĭm A K *et al* 1987 *Solid State Commun.* **10** 831
Bending S J and Geĭm A K 1992 *Phys. Rev.* B **46** 14 912
Geĭm A K *et al* 1992 *Phys. Rev.* B **46** 324
[2] Rammer J and Shelankov A L 1987 *Phys. Rev.* B **36** 3135
Bending S J, Klitzing K Von and Ploog K 1990 *Phys. Rev. Lett.* **65** 1060
[3] Geĭm A K, Bending S J and Grigorieva I V 1992 *Phys. Rev. Lett.* **69** 2252 and *Preprint*
[4] Khaetskiĭ A V 1991 *J. Phys.: Condens. Matter* **3** 5115
Kuptsov D A and Moiseev M Yu 1991 *J. Physique I* 1165
[5] Olariu S and Popescu I I 1985 *Rev. Mod. Phys.* **57** 339
[6] Abrikosov A A, Gorkov L P and Dzyaloshinski I E 1965 *Methods of Quantum Field Theory in Statistical Physics* (Oxford: Pergamon) ch 7 p 323 (Hall, New Jersey 1963)
[7] Iordanskiĭ S V 1965 *Zh. Teor. Fyz.* **49** 225 (Engl. transl. 1966 *JETP Lett.* **22** 160)
Iordanskiĭ S V and Koshelev A E 1986 *Zh. Teor. Fyz.* **90** 1399 (Engl. transl. 1986 *JETP Lett.* **63** 820)
[8] Aharonov Y and Bohm D 1959 *Phys. Rev.* **115** 485
[9] Sonin E B 1975 *Zh. Teor. Fyz.* **69** 921 (Engl. transl. 1975 *JETP Lett.* **42** 469)
[10] Aharonov Y *et al* 1984 *Phys. Rev.* D **29** 2396
[11] Geĭm A K and Bending S J Private communication
[12] For example in Weiss D *et al* 1991 *Phys. Rev. Lett.* **66** 2790; 1993 *Phys. Rev. Lett.* **70** 4118 and Weiss D Private communication
[13] Goldberg D Private communication
[14] The results of this letter were briefly presented in the 1993 March APS meeting (*Bull. APS* **38** 401 1993). We then received a preprint from L Brey and H A Fertig in which they have treated some aspects of the problem discussed here. However, they did not observe nor explain the rich structure of the electronic level degeneracies which are at the core of this letter.
[15] Carreau M, José J V and Rojas C *Preprint*
[16] Carreau M and José J V in preparation